Curso: Trigonometría

Ciclo Invierno 2020
TEMA N° 09

ECUACIONES TRIGONOMÉTRICAS

Son igualdades que contienen funciones de ángulos desconocidos que solamente se verifican para ciertos valores de esos ángulos que se llaman soluciones de la ecuación:

SOLUCIÓN BÁSICA (S.B)

Son aquellas que están comprendidas en [O; T], donde "T" es el período de la función.

OBSERVACIÓN

ANG. Δ REDUCIDO α	ANG. REDUCIDO β	
$\alpha \in$ IIC	$\beta = 180° - \alpha$	$\alpha = 180° - \beta$
$\alpha \in$ IIIC	$\beta = \alpha - 180°$	$\alpha = 180° + \beta$
$\alpha \in$ IVC	$\beta = 360° - \alpha$	$\alpha = 180° - \beta$

Donde : β ángulo de referencia.

EXPRESIÓN GENERAL DE TODOS LOS ÁNGULOS O ARCOS QUE TIENEN UNA MISMA FUNCIÓN TRIGONOMÉTRICA

Valor Principal (Vp):

1) E.G. PARA EL SENO Y COSECANTE

$$\boxed{X_G = k\pi + (-1)^K Vp = 180°k + (-1)^K Vp}$$

Donde : k: entero

2) E.G PARA EL COSENO Y SECANTE

$$\boxed{X_G = 2k\pi \pm Vp = 360°K \pm Vp}$$

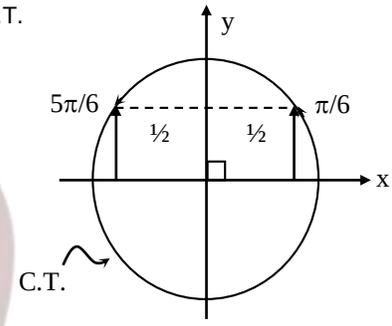
3) E.G PARA LA TANGENTE Y COTANGENTE

$$\boxed{X_G = K\pi + Vp = 180° + Vp}$$

PROBLEMAS RESUELTOS

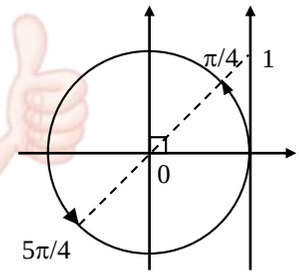
1) Resuelve : Sen x= $\frac{1}{2}$ para x ∈ [0; 2π]

Solución :
En la C.T.



Del gráfico las soluciones son : $\boxed{\frac{\pi}{6} \text{ y } \frac{5\pi}{6}}$

2) Resuelve : Tg2x = 1 para x ∈ [0; 2π]
Solución :



Luego :

$$2x = \frac{\pi}{4}; \frac{5\pi}{4}; \frac{9\pi}{4}; \frac{13\pi}{4}; \frac{17\pi}{4} \ldots$$

$$\rightarrow x = \frac{\pi}{8}; \frac{5\pi}{8}; \frac{9\pi}{8}; \frac{13\pi}{8}; \frac{17\pi}{8} \ldots$$

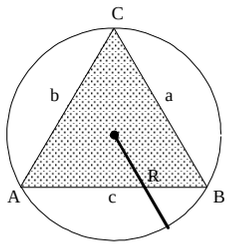
Tomando los valores que se encuentran en {0; 2π]

$$\boxed{\left\{\frac{\pi}{8}; \frac{5\pi}{8}; \frac{9\pi}{8}; \frac{13\pi}{8};\right\}}$$

RESOLUCIÓN DE TRIÁNGULOS OBLICUÁNGULOS

Triángulos Oblicuángulos

* **Ley de Senos**:

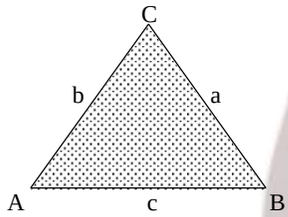


$$\frac{a}{\text{Sen}A} = \frac{b}{\text{Sen}B} = \frac{c}{\text{Sen}C} = 2R$$

2R : diámetro de la circunferencia circunscrita.

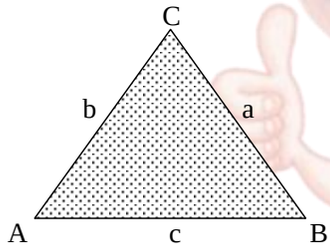
ó $\begin{cases} a = 2R\,\text{Sen}\,A \\ b = 2R\,\text{Sen}\,B \\ c = 2R\,\text{Sen}\,C \end{cases}$

* **Ley de Cosenos**:



$$\begin{cases} a^2 = b^2 + c^2 - 2bc\,\text{Cos}\,A \\ b^2 = a^2 + c^2 - 2ac\,\text{Cos}\,B \\ c^2 = a^2 + b^2 - 2ab\,\text{Cos}\,C \end{cases} \quad \text{ó} \quad \begin{cases} \text{Cos}\,A = \dfrac{b^2 + c^2 - a^2}{2bc} \\ \text{Cos}\,B = \dfrac{a^2 + c^2 - b^2}{2ac} \\ \text{Cos}\,C = \dfrac{a^2 + b^2 - c^2}{2ab} \end{cases}$$

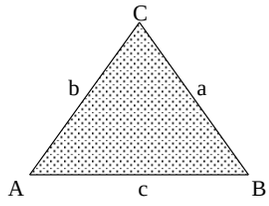
* **Ley de Tangentes**:



$$\frac{a+b}{a-b} = \frac{\text{Tg}\dfrac{A+B}{2}}{\text{Tg}\dfrac{A-B}{2}} \qquad \frac{a+c}{a-c} = \frac{\text{Tan}\left(\dfrac{A+C}{2}\right)}{\text{Tan}\left(\dfrac{A-C}{2}\right)}$$

$$\frac{b+c}{b-c} = \frac{\text{Tan}\left(\dfrac{B+C}{2}\right)}{\text{Tan}\left(\dfrac{B-C}{2}\right)}$$

* **Ley de proyecciones**:



$$\begin{aligned} a &= b\,\text{Cos}\,C + c\,\text{Cos}\,B \\ b &= a\,\text{Cos}\,C + c\,\text{Cos}\,A \\ c &= a\,\text{Cos}\,B + b\,\text{Cos}\,A \end{aligned}$$

Fórmulas de Briggs o de los semiángulos:

Senos

$$Sen\frac{A}{2} = \sqrt{\frac{(p-b)(p-c)}{bc}}$$

$$Sen\frac{B}{2} = \sqrt{\frac{(p-a)(p-c)}{ac}}$$

$$Sen\frac{C}{2} = \sqrt{\frac{(p-a)(p-b)}{ab}}$$

Cosenos

$$Cos\frac{A}{2} = \sqrt{\frac{p(p-a)}{bc}}$$

$$Cos\frac{B}{2} = \sqrt{\frac{p(p-b)}{ac}}$$

$$Cos\frac{C}{2} = \sqrt{\frac{p(p-c)}{ab}}$$

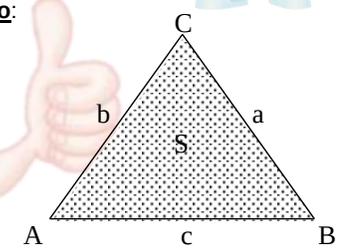
Tangentes

$$Tg\frac{A}{2} = \sqrt{\frac{(p-b)(p-c)}{p(p-a)}}$$

$$Tg\frac{B}{2} = \sqrt{\frac{(p-a)(p-c)}{p(p-b)}}$$

$$Tg\frac{C}{2} = \sqrt{\frac{(p-a)(p-b)}{p(p-c)}}$$

Área de un triángulo:



2p : perímetro
r : radio de la circunferencia inscrita.
r_a, r_b, r_c : radios de las circunferencias exinscritas relativas a los lados a, b, c respectivamente
R : radio de la circunferencia circunscrita

1) $S = \frac{ab}{2}\text{Sen}\,C = \frac{ac}{2}\text{Sen}B = \frac{bc}{2}\text{Sen}A$

2) $S = p.r$

3) $S = \frac{abc}{4R}$

4) $S = \sqrt{p(p-a)(p-b)(p-c)}$

5) $S = (p-a)r_a$
$S = (p-b)r_b$
$S = (p-c)r_c$
$S = \sqrt{r\ r_a r_b r_c}$

6) $S = 2R^2 \text{ Sen } A \text{ Sen } B \text{ Sen } C$

7) $S = r^2 \text{Cot}\frac{A}{2}\text{Cot}\frac{B}{2}\text{Cot}\frac{C}{2}$

8) $S = P^2 \text{Tan}\frac{A}{2}\text{Tan}\frac{B}{2}\text{Tan}\frac{C}{2}$

Elementos auxiliares:

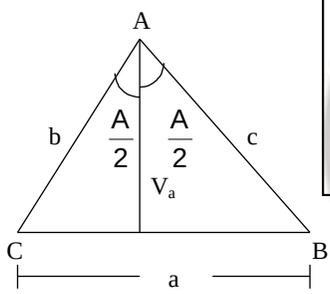
$$r = \sqrt{\frac{(p-a)(p-b)(p-c)}{P}} \quad r = (p-a)\text{Tg}\frac{A}{2}$$

$$r = (p-b)\text{Tg}\frac{B}{2}$$

$$r = (p-c)\text{Tg}\frac{C}{2} \qquad r_a = p\ \text{Tg}\frac{A}{2} \qquad r_b = p\text{Tg}\frac{B}{2}$$

$$r_c = p\text{Tg}\frac{C}{2}$$

Bisectrices interiores:

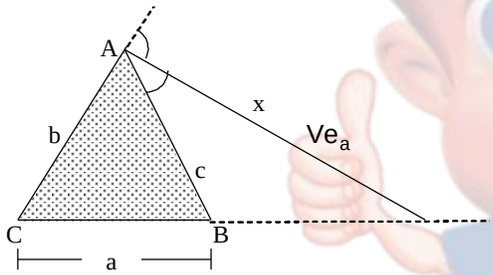


$$Vi_a = \frac{2bc}{b+c}\text{Cos}\frac{\hat{A}}{2}$$

$$Vi_b = \frac{2ac}{a+c}\text{Cos}\frac{\hat{B}}{2}$$

$$Vi_c = \frac{2ab}{a+b}\text{Cos}\frac{\hat{C}}{2}$$

Bisectrices exteriores:

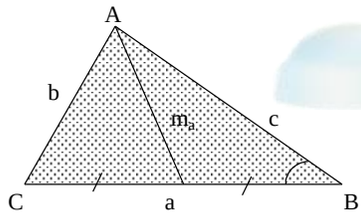


$$Ve_a = \frac{2bc}{|b-c|}\text{Sen}\frac{A}{2}$$

$$Ve_b = \frac{2ac}{|a-c|}\text{Sen}\frac{B}{2}$$

$$Ve_c = \frac{2ab}{|a-b|}\text{Sen}\frac{C}{2}$$

Mediana:

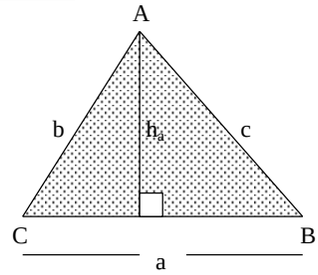


$$4m_a^2 = b^2 + c^2 + 2bc\text{Cos}\hat{A}$$

$$4m_b^2 = a^2 + c^2 + 2ac\text{Cos}\hat{B}$$

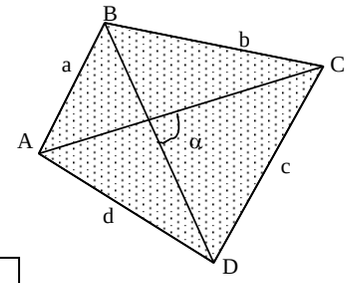
$$4m_c^2 = b^2 + c^2 + 2bc\text{Cos}\hat{C}$$

Alturas:



$h_A = c \text{ Sen } B$
$h_B = a \text{ Sen } C$
$h_C = b \text{ Sen } A$

Área de un cuadrilátero



$$S = \frac{AC \times BD}{2}\text{Sen}\alpha$$

$$S = \sqrt{(p-a)(p-b)(p-c)(p-d) - abcd\ \text{Cos}^2\frac{A+C}{2}}$$

→ Si es inscriptible

$$S = \sqrt{(p-a)(p-b)(p-c)(p-d)}$$

→ Si es circunscriptible

$$S = \sqrt{abcd\ \text{Sen}\frac{A+C}{2}}$$

→ Inscriptible y circunscriptible

$$S = \sqrt{abcd}$$